Chapter 1: Theory of Linear Capacitance
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Introduction:
Electrons have an electric field around them that originates from within each electron. This electric field causes electrons to repel each other or causes a proton and electron to attract each other. We use this property of “force at a distance” to store charge in capacitors and then use that stored charge to do work. Below is a description of linear capacitors, devices that are much simpler than the ferroelectric capacitors we will test with the RADIANT EDU. For a pictorial description of capacitance, open the presentation Simple Capacitance.

The Mechanism of Capacitance:
So, what is capacitance? It is a simple concept. Mark off a volume of space and store electrons in it. The electrons will repel each other and try to force each other back out of that volume. It takes force to make extra electrons go into that volume against their mutual repulsion. If we let them, the extra electrons will flow out. We can store work in that volume by forcing the electrons into the volume and get work out when we let the electrons flow out. The force required to store charge in the volume is the voltage on the capacitor. That voltage multiplied times the number of electrons tells us the energy stored in that volume. If we divide the number of electrons we forced into that volume by the voltage with which they are trying to get out, we get a ratio called the capacitance of that volume. In other words, if I use 1 volt to force in 10 electrons, I have a specific value of capacitance. If another volume will let me force in 20 electrons at 1V, then it has twice the capacitance. It is that simple!

Normally, a capacitor consists of two parallel plates of a conducting material separated by a distance. If the plates have a vacuum between them, then our ability to store the electrons on those plates is defined by the equation:

\[
\text{Capacitance} = \frac{\text{dielectric constant} \times \text{area}}{\text{thickness}} \quad \text{eq}(1.1)
\]

\[
\therefore \text{Capacitance} = C = \frac{\varepsilon_o \times A}{t} \quad \text{eq}(1.2)
\]

\(\varepsilon_o\) = electrical dielectric constant of a vacuum
\(\varepsilon_o = 8.84 \times 10^{-14}\) Farads/cm
A = capacitor area in square centimeters
t = capacitor thickness in centimeters
There is geometry associated with the capacitor and its properties. Note that the larger the area of the plates, the larger the capacitance. The smaller the distance between the plates, the larger the capacitance. The one parameter in equation (1.2) that is independent of the geometry is the fundamental physical constant “$\varepsilon_0$”, called the dielectric constant. The parameter $\varepsilon_0$ tells us how hard the electrons repel each other. “$\varepsilon_0$” is a very powerful parameter, one that organizes the universe. For a discussion of this wonderful physical constant, read Chapter 2 - The Importance of Dielectric Constant.

Using this new parameter “$C$”, the number of electrons stored in the capacitor at a voltage “$V$” is

$$\#\text{electrons} \quad = \quad N \cdot e \quad = \quad Q \quad = \quad C \cdot \text{Volts} \quad \quad \text{eq(1.3)}$$

- $e$ = the charge of one electron
- $N$ = the number of electrons
- $Q$ = the total charge in units of Coulombs.

A Coulomb is a measure of the number of electrons. One Coulomb of charge is $6.25 \times 10^{18}$ electrons. Therefore, one electron, “$e$” in equation (1.3), is $1.6 \times 10^{-19}$ Coulombs. A Farad is the measure of capacitance. It is defined such that

$$1 \text{ Farad} \quad = \quad 1 \text{ Coulomb} / 1 \text{ Volt} \quad \quad \text{eq(1.4)}$$

With the RADIANT EDU tester, charge is given in units of microCoulombs and capacitance in units of microFarads or nanoFarads or picoFarads. The meanings of the metric prefixes are listed below.

- milli = 1 thousandth = $1 \times 10^{-3}$
- micro = 1 millionth = $1 \times 10^{-6}$
- nano = 1 billionth = $1 \times 10^{-9}$
- pico = 1 trillionth = $1 \times 10^{-12}$
- femto = 1 quadrillionth = $1 \times 10^{-15}$
- atto = 1 quintillionth = $1 \times 10^{-18}$

To give you an idea of what this means:
The transistors in your desktop computer have about one femtoFarad of capacitance each. So, at three volts it takes about 3 femtoCoulombs to turn one ON or OFF.

Those same transistors are less than one half of a micrometer in length, which is one half of one millionth of a meter. This length can also be described as 500 nanometers.

An adenovirus, the type of virus that can gives you a cold, is about 100 nanometers across.

Some living cells are about 1 micrometer across. Some are smaller. Others are larger.

A human hair is on average about 100 micrometers in diameter.

The standard credit card is roughly 750 micrometers, or 0.75 millimeters, in thickness.

The standard ferroelectric capacitor supplied with the RADIANT EDU is 120 micrometers long on one side and 80 micrometers long on the other to give it an area of 10,000 square micrometers. This is equivalent to 0.0001 square millimeters. The capacitor is 260 nanometers thick and has roughly 1 nanoFarad in total capacitance.

The leads on the TO-18 package of the Radiant ferroelectric capacitors are 0.2 millimeters in diameter and have a capacitance of roughly 1 picoFarad by themselves.

NOTE: It is common in the research community to refer to a “micrometer” as a “micron”.

According to equation (1.3) above, if we apply 1 volt to a 1 nanoFarad capacitor, we will store 1 nanoCoulomb in that capacitor. To give you an understanding of how small this amount of charge is, a standard electric kitchen stove will use about 20 amperes of current to heat the element. An ampere is the flow of one Coulomb of charge per second through the circuit. So, the kitchen stove uses 20 Coulombs per second to heat the burner. We, on the other hand, will store only 1 nanoCoulomb in one of our ferroelectric capacitors during a hysteresis test. Your desktop computer will use only 3 femtoCoulombs to turn ON or OFF a single transistor during a calculation by the personal computer.
20 Coulombs = 20 \cdot 6.25 \times 10^{18} \text{ electrons} = 1.25 \times 10^{20} \text{ electrons}

1 \text{ nanoCoulomb} = (1 \times 10^{-9}) \cdot (6.25 \times 10^{18} \text{ electrons}) = 6.25 \text{ billion electrons}

1 \text{ femtoCoulomb} = (1 \times 10^{-15}) \cdot (6.25 \times 10^{18} \text{ electrons}) = 6250 \text{ electrons}

The transistor operation seems to take almost no energy. But, remember that a desktop computer chip may have 100,000,000 (or 1\times10^8) transistors turning off and on at 100,000,000 (or 1\times10^7) times per second each on average. (Some are turning ON/OFF every nanosecond and some sit around a lot.) So, the computer chip could use as much as

\[
\text{Current} = \frac{\text{Total Charge}}{\text{Time}} = \text{Number of transistors in the computer} \times \text{rate at which each transistor turns ON or OFF per second} \times 3\text{femtoCoulombs per ON/OFF} \times \#\text{electrons/Coulomb}
\]

\[
\text{Current} = (1\times10^8) \cdot (1\times10^8) \cdot (3\times10^{-15}) \cdot (6.25\times10^{18} \text{ electrons}) \text{ per second} = 1.875 \times 10^{20} \text{ electrons per second} = 30 \text{ amps}
\]

The modern desktop computer chip would melt if it did not have its own fan. Some must now be water cooled.

The point here is that capacitance is strictly about keeping track of how many electrons move into and out of the capacitor at each change in the voltage across the capacitor. Ferroelectric capacitors are unique because not all of the electrons that go in come out; this property gives the ferroelectric capacitor memory.

**Relative Dielectric Constant:**
All of the discussion so far has been about the how a vacuum capacitor works. You can increase its capacitance by putting material inside the capacitor between the plates. A vacuum has capacitance that we have arbitrarily assigned the “relative” value as “1”. If we fill the vacuum between the plates with any material whatsoever, the capacitance goes up. This means that you will be able to stuff more electrons into that capacitor at a
specified voltage than you could have if you had left a vacuum in the capacitor. (See Simple Capacitance for a pictorial explanation of this phenomenon.)

The reason is that almost all matter at room temperature or body temperature consists of electrons paired with protons in atoms and molecules. As you charge up the capacitor, the molecules and atoms inside the capacitor see the lines of force, the electric field, emanating from the charges you forced into capacitor. These lines of force cause the electrons in the molecule or atom to try to go in one direction and the protons to go in the other. (Of course, we do not want to rip electrons from the protons so we use low voltages on capacitors. Lightening is what happens in nature when the “ripping” voltage is exceeded.) The net effect is that all charges inside the atom or molecule become separated by a short distance, creating a tiny little nano-capacitor out of that atom or molecule. These nano-capacitors, trillions of them, are aligned opposite to the direction of our big capacitor so they cancel out some of its effect. This allows us to force more charge into the big capacitor at the same voltage. Each material we put between the plates has a different effect. The ratio of the number of electrons that we can force into a capacitor with and without a material inside the capacitor is called the “relative dielectric constant” ($\varepsilon_r$). If $\varepsilon_r$ is 2, then we can force twice as many electrons into the capacitor at the same voltage as we could if there was only a vacuum in the capacitor. The fundamental dielectric constant of the universe, of a vacuum, is $\varepsilon_0$. The parameters $\varepsilon_0$ and $\varepsilon_r$ together tell us the true capacitance of the device.

$$\varepsilon_{\text{total}} = \varepsilon = \varepsilon_0 \cdot \varepsilon_r \quad \text{eq}(1.5)$$

**One Common Application of Capacitors Used Every Day**

You can store information in a capacitor. A capacitor can be either charged or empty, indicating a “1” or a “0” in the binary language of computers. To read it, though, you have to discharge it to see if any charge is there. This destroys the data and you have to write the data back after the read operation is over. The data is “dynamic”. In fact, Dynamic Random Access Memories (DRAMs) are the foundation of modern computers. When you order your new computer with 1 gigabyte of main memory, you are actually telling the manufacturer to put

$$1024 \cdot (1024 \times 1024) \cdot 8 = 8,589,934,592$$

-> A “bit” is a single “1” or “0” datum.
-> There are eight bits of data in a byte.
-> A megabyte is $1024 \times 1024$ bytes.
tiny capacitors in your computer to hold your programs and data while the applications run. The amazing thing about these capacitors is that they are only about 20 atoms thick [see eq(1.1)], every chip has about a billion of these little guys, and none of them have holes in them. Imagine the quality control the semiconductor industry must apply to make these memories work at so little cost to you the consumer.

**Linearity**

The capacitors described so far are called *linear* capacitors. The reason is that when we substitute equation (1.2) into equation (1.3), all of the parameters are constant and we get the equation of a line.

\[
Q = \varepsilon_0 \cdot (A / t) \cdot \text{Volts} \tag{eq(1.6)}
\]
\[
Y = mX + b \tag{eq(1.7)}
\]
\[
Y = Q = C \cdot V \tag{eq(1.3)}
\]

- \(m = C\) = slope of the line
- \(b = \) the Y-intercept point is “0” because the capacitor always discharges to 0 volts if it can.

So, if we apply a range of voltages to a linear capacitor, measure how much charge goes in or out, and plot the results, we should get a straight line according to eq(1.6).

The measurement of a 1.0nF ceramic disk capacitor from Radio Shack on the RADIANT EDU tester shows that it is indeed a linear device.
Let’s see if the capacitance label on this device is correct. The capacitance label states that it is a 1.0nF capacitor. The measurement by the RADIANT EDU shows approximately 0.005 microcoulombs, which is 5.0 nanocoulombs. This charge was generated at 5.0V. So, the capacitance is

\[
\text{Capacitance} = \frac{\text{Charge}}{\text{Voltage}}
\]

\[
= \frac{5.0\text{nC}}{5.0\text{V}}
\]

\[
= 1.0\text{nF}.
\]

Note that the vertical axis of this measurement is in units of charge. Normally, ferroelectric testers divide the measured charge by the area of the capacitor and plot this result in units of “polarization”. By doing this, capacitors of different sizes can be compared on the same graph. For instance, a 1.0nF capacitor at 1.0V will generate 1.0nC. A 1.0µF capacitor (microfarad) at 1.0V will generate 1.0µC of charge (microCoulomb). If we plot these two on the same graph, we will see the data for the 1.0µF capacitor but the data for the 1.0nF capacitor, being 1000 times smaller, will be in the X-axis of the graph. If the capacitors are made of the same material so they have the same \(\varepsilon\) and they are the same thickness, then the only difference between them will be their respective areas. If we divide the measured charge of both capacitors by their respective area, then the two measurements should lie on top of each other in the graph. When you divide the
charge by the area it came from, you get units of “Polarization” which, on the RADIANT EDU default to

\[ \mu \text{C/cm}^2 \]

or microcoulombs per square centimeter. The symbol “\( \mu \)” means “micro”.

(NOTE: For those of you that have already tried the EDU or have used research level testers, you can plot “charge” with the EDU instead of “polarization” by setting the area of the capacitor to 1.0cm\(^2\) so the measured charge is divided by “1” before plotting.)

Capacitors such as the 1.0nF disk capacitor in Figure 1.1 are used by the billions around the world each year to make computers, cars, watches, TVs, radios, stoves, washing machines, and on and on. The capacitors we are interested in are non-linear capacitors. A non-linear capacitor does not make a straight line on the charge vs. voltage measurement. Non-linear capacitors, and especially ferroelectric capacitors, are the subjects of the third lesson.

The next chapter, Chapter 2 - The Importance of Dielectric Constant, is a description of how the dielectric constant shapes the universe we live in. The purpose of Chapter 2 is to give you an understanding of how capacitance is intrinsically woven into the fabric of our very existence. It is not necessary for you to read and understand Chapter 2 to understand the remainder of the lessons. You may skip it and proceed directly to Chapter 3 - Paraelectric Capacitors if you wish. I personally recommend that you read Chapter 2 for the fun of it.