Ferroelectric Circuit Equations

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Why Circuit Equations?

FeRAM capacitors operate in a perfect world:

1) one side tied to ground,
2) the other to voltage, and
3) always starting in one of two saturated states.

Curve fitting will do.

Inside an analog circuit, the voltage across a ferroelectric capacitor depends upon the other components which depend upon the ferroelectric capacitor. A functional model of the ferroelectric capacitor is required!
Why Circuit Equations?

In an analog circuit, a ferroelectric capacitor can take on analog states:

- The charge state of the capacitor is not unique given a voltage condition.
- The voltage state is unique given an initial charge condition.
Derivation

- **Goal** – Introduce ferroelectric behavior to the following equation:

  \[ I = C \frac{dV}{dt} \]

- **Procedure**:
  - Define the charge equation for the ferroelectric capacitor.
  - Take the derivative with respect to voltage.
  - Substitute \( C_{FE} \) for \( C \) in the dynamic equation.
Ferroelectric Charge

- A ferroelectric capacitor can be defined as seven charge sources in parallel:
  1) Linear charge
  2) Paraelectric charge
  3) Remanent charge
  4) Polarization modulated resistive leakage
  5) Polarization modulated capacitance charge
  6) Surface contact diode leakage
  7) Other sources such as internal mobile space charge.
Ferroelectric Charge

- A well-behaved ferroelectric capacitor can be mostly defined using the four largest contributors to the shape of its hysteresis loop:

1) Linear charge
2) Paraelectric charge
3) Remanent charge
4) Polarization modulated resistive leakage
5) Polarization modulated capacitance charge
6) Surface contact diode leakage
7) Other sources such as internal mobile space charge
Ferroelectric Components

**Linear Charge**

**Remanent Charge**

**Paraelectric + Diode Charge**
Ferroelectric Components

Capacitance is the derivative of the charge loop!

Linear Charge

Remanent Charge

Paraelectric + Diode Charge
Surface Contact Diode

- The surface contact diode creates the open tip of the loop and the open tail of the capacitance loop.
Hysteresis to CvsV
Ferroelectric Components

- The derivative of a loop along with hand fitted components in color.
Capacitance Equations

- Linear Capacitor: \( C = constant \)
- Paraelectric Capacitor:
  \[
  C_{\text{Paraelectric}} = \frac{C_{\text{Paraelectric}}}{\sigma_{\text{Paraelectric}} \sqrt{2\pi}} e^{-\left[\frac{v}{2\sigma_{\text{Paraelectric}}}\right]^2}
  \]
- Diode Capacitor:
  \[
  C = [ +constant ] \text{ for increasing voltage}
  \]
  \[
  C = [ -constant ] \text{ for increasing voltage}
  \]
  \[
  M = 1 \text{ or } 0
  \]
Capacitance Equations

- Remanent Capacitor:

\[
C = \frac{C_{Rem}}{\sigma_{Rem}^{+} \sqrt{2\pi}} M^{+} e^{-\left[\frac{(\nu - \nu_{c}^{+})}{2\sigma_{Rem}^{+}}\right]^{2}} + \frac{C_{Rem}}{\sigma_{Rem}^{-} \sqrt{2\pi}} M^{-} e^{-\left[\frac{(\nu - \nu_{c}^{-})}{2\sigma_{Rem}^{-}}\right]^{2}}
\]

where \( M = 1 \) or 0
$C_{FE}$ Equation

- Towards $V_{Max}$:

$$C_{FE} = C_{Linear} + \frac{C_{Paraelectric}}{\sigma_{Paraelectric}\sqrt{2\pi}} e^{-\left[\frac{V-V_{Offset}}{2\sigma_{Paraelectric}}\right]^2} + M \frac{C_{Remanent}}{\sigma_{Remanent}\sqrt{2\pi}} e^{-\left[\frac{(V-V_{c+})}{2\sigma_{Remanent}}\right]^2} + C_{Diode}$$

$M = 1$ for switching or
$M = 0$ for non-switching

- Return-to-Zero:

$$C_{FE} = C_{Linear} + \frac{C_{Paraelectric}}{\sigma_{Paraelectric}\sqrt{2\pi}} e^{-\left[\frac{V-V_{Offset}}{2\sigma_{Paraelectric}}\right]^2} - C_{Diode}$$
Fitted Components

Type AD103 & Components

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{Linear}$</td>
<td>0.3nF</td>
</tr>
<tr>
<td>$C_{Paraelectric}$</td>
<td>0.7nF</td>
</tr>
<tr>
<td>$\sigma_{Paraelectric}$</td>
<td>0.8v</td>
</tr>
<tr>
<td>$C_{Remanent}$</td>
<td>2.3nF</td>
</tr>
<tr>
<td>$\sigma_{Remanent}$</td>
<td>0.23v</td>
</tr>
<tr>
<td>$+V_{Coercive}$</td>
<td>1.66v</td>
</tr>
<tr>
<td>$C_{diode}$</td>
<td>0.075nF</td>
</tr>
</tbody>
</table>

- The derivative of a loop along with hand fitted components in color.
Fitted Components

- Summation of the four fitted components.
Fitted Components w/o Surface Diode

- The surface contact diode was not included in the circuit model.
Simple RC Circuit

\[ V_{Out} = V_{PWR} \left( 1 - e^{-\frac{t}{RC}} \right) \]

\[ V_{FE} = V_{PWR} \left( 1 - e^{-\frac{t}{RC_{FE}}} \right) \]
Simple RC Circuit

- The rate at which the exponential term goes to zero in time is controlled by the magnitude of \( C_{FE} \).

  \[ \text{Larger } C_{FE} \Rightarrow \text{slower rise in } V_{out} \]
  \[ \text{Smaller } C_{FE} \Rightarrow \text{faster rise in } V_{out} \]

- Initial: \( C_{FE} = C_{\text{Linear}} + C_{\text{Paraelectric}} + C_{\text{Diode}} \)
- Near \( V_C \): \( C_{FE} = C_{\text{Linear}} + C_{\text{Paraelectric}} + C_{\text{Diode}} + C_{\text{remanent}} \)
- Near \( V_P \): \( C_{FE} = C_{\text{Linear}} + C_{\text{Paraelectric}} + C_{\text{Diode}} \)
Simple RC Circuit

- Type AD = 3/20/80 PNZT
Simple RC Circuit

- Simulated model (red and blue) versus measurement of real component (black dashed).
Simple RC Circuit

- Type AD = 3/20/80 PNZT.  \( R_{\text{Series}} = 1\,\text{M}\Omega \).
Autonomous Memory Circuit

\[ V_{Out} = V_p \left( 1 - e^{\frac{-t}{(1+\beta)RC_{FE}}} \right) \]
Autonomous Memory Circuit

RCX: \[ V_{out} = V_p \left( 1 - e^{(1+\beta)RC_{FE}} \right) \]
RC: \[ V_{out} = V_p \left( 1 - e^{RC_{FE}} \right) \]

Comparison of measurements of RC circuit vs RC-Transistor circuit if RC circuit time scale is stretched by the value \( \beta = 2 \).

- Type AB = 20/80 PZT
Conclusions

- The mathematical derivative of the ferroelectric hysteresis loop yields the effective capacitance of the FeCap.
- The capacitance curve can be fit for the different components known to constitute the hysteresis loop.
- The sum of the ferroelectric capacitance components may be substituted for the linear capacitor in classic circuit equations.
- *The model does a reasonable job of predicting circuit behavior but the accuracy will depend upon how many components are included in the sum and how accurately each component is modeled.*